STUDENT #: _______________
NAME: ____________________

ASSIGNMENT 8:
Wave Optics

UNIVERSITY OF OTTAWA
Principles of Physics
PHY1322 Winter 2017

Released: **March 10 Due: March 16 6PM sharp!**
(32 points total)

1 The Impressionist painter Georges Seurat created paintings with an enormous number of dots of pure pigment, each of which was approximately 2.00 mm in diameter. The idea was to have colors such as red and green next to each other to form a scintillating canvas (Fig. P38.17). Outside what distance would one be unable to discern individual dots on the canvas? (Assume that λ = 500 nm and that the pupil diameter is 4.00 mm.)

By Rayleigh's criterion, two dots separated center-to-center by 2.00 mm would overlap

when $\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$.

Thus, $L = \frac{dD}{1.22\lambda} = \frac{(2.00\times10^{-3}\text{ m})(4.00\times10^{-3}\text{ m})}{1.22(500\times10^{-9}\text{ m})} = \boxed{13.1\text{ m}}$.

2A Grote Reber was a pioneer in radio astronomy. He constructed a radio telescope with a 10.0-m-diameter receiving dish. What was the telescope's angular resolution for 2.00-m radio waves?

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{(2.00\text{ m})}{(10.0\text{ m})} = \boxed{0.244\text{ rad} = 14.0°}$$

2B The pupil of a cat's eye narrows to a vertical slit of width 0.500 mm in daylight. What is the angular resolution for horizontally separated mice? Assume that the average wavelength of the light is 500 nm.

$$\sin\theta = \frac{\lambda}{a} = \frac{5.00\times10^{-7}\text{ m}}{5.00\times10^{-4}} = \boxed{1.00\times10^{-3}\text{ rad}}$$

3A A beam of green light is diffracted by a slit of width 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 4.10 mm. Calculate the wavelength of the laser light.

The positions of the first-order minima are $\frac{y}{L} \approx \sin\theta = \pm\frac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y = 2\left(\frac{\lambda}{a}\right)L$ and the wavelength is $\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10\times10^{-3}\text{ m}}{2}\right)\left(\frac{0.550\times10^{-3}\text{ m}}{2.06\text{ m}}\right) = \boxed{547\text{ nm}}$.

3B A screen is placed 50.0 cm from a single slit, which is illuminated with 690-nm light. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

$\frac{y}{L} \approx \sin\theta = \frac{m\lambda}{a}$ $\Delta y = 3.00\times10^{-3}\text{ nm}$ $\Delta m = 3-1 = 2$ and $a = \frac{\Delta m\lambda L}{\Delta y}$ $a = \frac{2(690\times10^{-9}\text{ m})(0.500\text{ m})}{(3.00\times10^{-3}\text{ m})} = \boxed{2.30\times10^{-4}\text{ m}}$

4A Light from an argon laser strikes a diffraction grating that has 5 310 grooves per centimeter. The central and firstorder principal maxima are separated by 0.488 m on a wall 1.72 m from the grating. Determine the wavelength of the light. laser

4A

The principal maxima are defined by

$$d\sin\theta = m\lambda \qquad m = 0,\ 1,\ 2,\ \ldots$$

For $m = 1$, $\qquad \lambda = d\sin\theta$

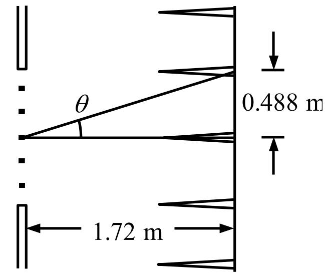

FIG. P38.24

where θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

$$\tan\theta = \frac{0.488\ \text{m}}{1.72\ \text{m}} = 0.284 \quad \text{so} \qquad \theta = 15.8° \quad \text{and} \qquad \sin\theta = 0.273.$$

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$$d = \frac{1}{5\,310\ \text{cm}^{-1}} = 1.88\times10^{-4}\ \text{cm} = 1.88\times10^{3}\ \text{nm}.$$

The wavelength is $\qquad \lambda = d\sin\theta = \left(1.88\times10^{3}\ \text{nm}\right)(0.273) = \boxed{514\ \text{nm}}$.

4B In a double slit experiment with the coherent light source of 500nm, d= 3a. Identify missing interference maxima in the diffraction/interference pattern on the screen.

$$\frac{d\ \sin\theta}{a\ \sin\theta} = \frac{m\lambda}{\lambda} \rightarrow \frac{d}{a} = m \qquad \text{m=3}$$

5A The human eye is most sensitive to 560-nm light. What is the temperature of a black body that would radiate most intensely at this wavelength?

$$\lambda_{\max} T = 2.898\times10^{-3}\, m.K \ \text{ so } \ \text{T=5175K}$$

5B The radius of our Sun is 6.96 × 10^{8} m, and its total power output is 3.77 × 10^{26} W. Assuming that the Sun's surface emits as a black body, calculate its surface temperature. And find λ_{max} for the Sun.

$$\text{P} = eA\sigma T^4, \text{so } T = \left(\frac{\text{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.77\times10^{26}\ \text{W}}{1\left[4\pi\left(6.96\times10^{8}\ \text{m}\right)^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{5.75\times10^{3}\ \text{K}}$$

(b) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.75\times10^{3}\ \text{K}} = 5.04\times10^{-7}\ \text{m} = \boxed{504\ \text{nm}}$$

6 (8p) The total power per unit area radiated by a black body at a temperature T is the area under the $I(\lambda, T)$-versus-λ curve, as shown in Figure 40.3. (a) Show that this power per unit area is

$$\int_0^\infty I(\lambda, T)\,d\lambda = \sigma T^4$$

where $I(\lambda, T)$ is given by Planck's radiation law and σ is a constant independent of T. This result is known as Stefan's law. (See Section 20.7.) To carry out the integration, you should make the change of variable $x = hc/\lambda kT$ and use the fact that

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

(b) Show that the Stefan–Boltzmann constant σ has the value

$$\sigma = \frac{2\pi^5 k_B{}^4}{15c^2h^3} = 5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}$$

Starting with Planck's law, $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}$

the total power radiated per unit area $\displaystyle\int_0^\infty I(\lambda, T)\,d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}\,d\lambda$.

Change variables by letting $x = \dfrac{hc}{\lambda k_B T}$

and $dx = -\dfrac{hc\,d\lambda}{k_B T\lambda^2}$.

Note that as λ varies from $0 \to \infty$, x varies from $\infty \to 0$.
Then

$$\int_0^\infty I(\lambda, T)\,d\lambda = -\frac{2\pi k_B^4 T^4}{h^3c^2}\int_\infty^0 \frac{x^3}{\left(e^x - 1\right)}\,dx = \frac{2\pi k_B^4 T^4}{h^3c^2}\left(\frac{\pi^4}{15}\right).$$

Therefore, $\boxed{\displaystyle\int_0^\infty I(\lambda, T)\,d\lambda = \left(\frac{2\pi^5 k_B^4}{15h^3c^2}\right)T^4 = \sigma T^4}$.

(b) From part (a),

$$\sigma = \frac{2\pi^5 k_B^4}{15h^3c^2} = \frac{2\pi^5\left(1.38\times10^{-23}\ \mathrm{J/K}\right)^4}{15\left(6.626\times10^{-34}\ \mathrm{J\cdot s}\right)^3\left(3.00\times10^8\ \mathrm{m/s}\right)^2}$$

$$\sigma = \boxed{5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}}.$$